

5.254 orchard

	DESCRIPTION	LINKS	GRAPH
Origin	[198]		
Constraint	orchard(NROW, TREES)		
Arguments	NROW : dvar TREES : collection(index-int, x-dvar, y-dvar)		
Restrictions	NROW \geq 0 TREES.index \geq 1 TREES.index \leq TREES required(TREES, [index, x, y]) distinct(TREES, index) TREES.x \geq 0 TREES.y \geq 0		
Purpose	Orchard problem [198]: <i>“Your aid I want, Nine trees to plant, In rows just half a score, And let there be, In each row, three—Solve this: I ask no more!”</i>		
Example	$10, \left(\begin{array}{l} \text{index} - 1 \quad x - 0 \quad y - 0, \\ \text{index} - 2 \quad x - 4 \quad y - 0, \\ \text{index} - 3 \quad x - 8 \quad y - 0, \\ \text{index} - 4 \quad x - 2 \quad y - 4, \\ \text{index} - 5 \quad x - 4 \quad y - 4, \\ \text{index} - 6 \quad x - 6 \quad y - 4, \\ \text{index} - 7 \quad x - 0 \quad y - 8, \\ \text{index} - 8 \quad x - 4 \quad y - 8, \\ \text{index} - 9 \quad x - 8 \quad y - 8 \end{array} \right)$		
	The 10 alignments of 3 trees correspond to the following triples of trees: (1, 2, 3), (1, 4, 8), (1, 5, 9), (2, 4, 7), (2, 5, 8), (2, 6, 9), (3, 5, 7), (3, 6, 8), (4, 5, 6), (7, 8, 9). Figure 5.474 shows the 9 trees and the 10 alignments corresponding to the example.		
Symmetries	<ul style="list-style-type: none"> • Items of TREES are permutable. • Attributes of TREES are permutable w.r.t. permutation (index) (x, y) (<i>permutation applied to all items</i>). • One and the same constant can be added to the x attribute of all items of TREES. • One and the same constant can be added to the y attribute of all items of TREES. 		
Keywords	characteristic of a constraint: hypergraph. geometry: geometrical constraint, alignment.		

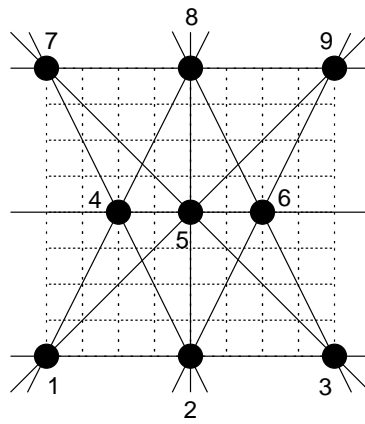


Figure 5.474: Nine trees with 10 alignments of 3 trees

Arc input(s)	TREES
Arc generator	<i>CLIQUE</i> (\langle) \mapsto <code>collection</code> (trees1, trees2, trees3)
Arc arity	3
Arc constraint(s)	$\sum \begin{pmatrix} \text{trees1.x} * \text{trees2.y} - \text{trees1.x} * \text{trees3.y}, \\ \text{trees1.y} * \text{trees3.x} - \text{trees1.y} * \text{trees2.x}, \\ \text{trees2.x} * \text{trees3.y} - \text{trees2.y} * \text{trees3.x} \end{pmatrix} = 0$
Graph property(ies)	<u>NARC</u> = NROW

Graph model

The arc generator *CLIQUE*(\langle) with an arity of three is used in order to generate all the arcs of the directed hypergraph. Each arc is an ordered triple of trees. We use the restriction \langle in order to generate one single arc for each set of three trees. This is required, since otherwise we would count more than once a given `alignment` of three trees. The formula used within the arc constraint expresses the fact that the three points of respective coordinates (`trees1.x`, `trees1.y`), (`trees2.x`, `trees2.y`) and (`trees3.x`, `trees3.y`) are aligned. It corresponds to the development of the expression:

$$\begin{vmatrix} \text{trees1.x} & \text{trees2.y} & 1 \\ \text{trees2.x} & \text{trees2.y} & 1 \\ \text{trees3.x} & \text{trees3.y} & 1 \end{vmatrix} = 0$$

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