5.202 link_set_to_booleans

DESCRIPTION LINKS GRAPH

Origin Inspired by domain_constraint.

Constraint link_set_to_booleans(SVAR, BOOLEANS)

Arguments

SVAR	:	svar
BOOLEANS	:	collection(bool−dvar, val−int)

Restrictions

required(BOOLEANS, [bool, val])
BOOLEANS.bool ≥ 0
BOOLEANS.bool ≤ 1
distinct(BOOLEANS, val)

Purpose Make the link between a set variable SVAR and those 0-1 variables that are associated with each potential value belonging to SVAR: The 0-1 variables, which are associated with a value belonging to the set variable SVAR, are equal to 1, while the remaining 0-1 variables are all equal to 0.

Example

$$\left(\begin{array}{l} \{1,3,4\}, \\ \left\langle \begin{array}{ll} \texttt{bool}-0 & \texttt{val}-0, \\ \texttt{bool}-1 & \texttt{val}-1, \\ \texttt{bool}-0 & \texttt{val}-2, \\ \texttt{bool}-1 & \texttt{val}-3, \\ \texttt{bool}-1 & \texttt{val}-4, \\ \texttt{bool}-0 & \texttt{val}-5 \end{array} \right\rangle \end{array}\right)$$

In the example, the 0-1 variables associated with the values 1, 3 and 4 are all set to 1, while the other 0-1 variables are set to 0. Consequently, the link_set_to_booleans constraint holds since its first argument SVAR is set to $\{1, 3, 4\}$.

Symmetry Items of BOOLEANS are permutable.

Usage This constraint is used in order to make the link between a formulation using set variables and a formulation based on linear programming.

Systems channel in **Gecode**.

See also **common keyword:** alldifferent_between_sets, clique (*constraint involving set variables*), domain_constraint (*channelling constraint*), k_cut, path_from_to, roots, strongly_connected, symmetric_cardinality, symmetric_gcc, tour (*constraint involving set variables*).

Keywords **characteristic of a constraint:** derived collection.

constraint arguments: constraint involving set variables.

constraint type: decomposition, value constraint.

filtering: linear programming.

modelling: channelling constraint, set channel.

Derived Collection

$$\texttt{col}\left(\begin{array}{l}\texttt{SET}-\texttt{collection}(\texttt{one}-\texttt{int},\texttt{setvar}-\texttt{svar}),\\ [\texttt{item}(\texttt{one}-1,\texttt{setvar}-\texttt{SVAR})]\end{array}\right)$$

Arc input(s)	SET BOOLEANS
Arc generator	$PRODUCT \mapsto \texttt{collection}(\texttt{set},\texttt{booleans})$
Arc arity	2
Arc constraint(s)	$\texttt{booleans.bool} = \texttt{set.one} \Leftrightarrow \texttt{in\_set}(\texttt{booleans.val},\texttt{set.setvar})$
Graph property(ies)	$\mathbf{NARC} = \lvert\texttt{BOOLEANS}\rvert$

Graph model

The link_set_to_booleans constraint is modelled with the following bipartite graph. The first set of vertices corresponds to one single vertex containing the set variable. The second class of vertices contains one vertex for each item of the collection BOOLEANS. The arc constraint between the set variable SVAR and one potential value v of the set variable expresses the following:

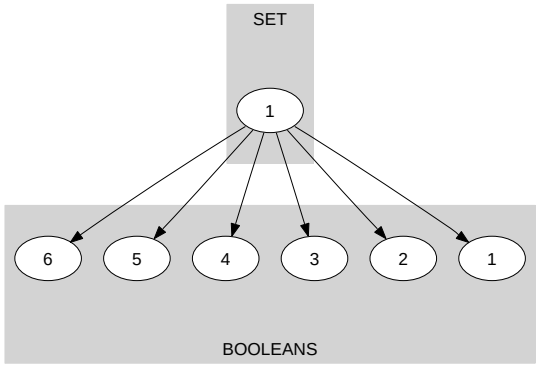
- If the 0-1 variable associated with v is equal to 1 then v should belong to SVAR.
- Otherwise if the 0-1 variable associated with v is equal to 0 then v should not belong to SVAR.

Since all arc constraints should hold the final graph contains exactly |BOOLEANS| arcs.

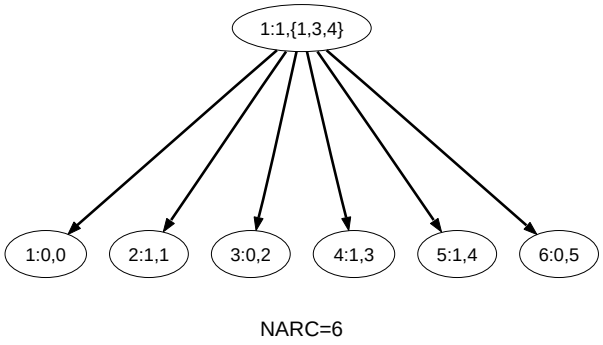
Parts (A) and (B) of Figure 5.395 respectively show the initial and final graph associated with the **Example** slot. Since we use the **NARC** graph property, the arcs of the final graph are stressed in bold. The link_set_to_booleans constraint holds since the final graph contains exactly 6 arcs (one for each 0-1 variable).

Signature

Since the initial graph contains |BOOLEANS| arcs the maximum number of arcs of the final graph is equal to |BOOLEANS|. Therefore we can rewrite the graph property $\mathbf{NARC} = |\texttt{BOOLEANS}|$ to $\mathbf{NARC} \geq |\texttt{BOOLEANS}|$ and simplify $\underline{\overline{\mathbf{NARC}}}$ to $\overline{\mathbf{NARC}}$.



(A)



(B)

Figure 5.395: Initial and final graph of the `link_set_to_booleans` constraint