5.145 graph_crossing

DESCRIPTION LINKS GRAPH

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Constraint graph_crossing(NCROSS, NODES)

Synonyms crossing, ncross.

Arguments

NCROSS	:	dvar
NODES	:	collection(succ−dvar, x−int, y−int)

Restrictions

$\texttt{NCROSS} \geq 0$
required(NODES, [succ, x, y])
$\texttt{NODES.succ} \geq 1$
$\texttt{NODES.succ} \leq |\texttt{NODES}|$

Purpose NCROSS is the number of proper intersections between line-segments, where each line-segment is an arc of the directed graph defined by the arc linking a node and its unique successor.

Example

$$\left(2, \left\langle \begin{array}{lll} \texttt{succ}-1 & \texttt{x}-4 & \texttt{y}-7, \\ \texttt{succ}-1 & \texttt{x}-2 & \texttt{y}-5, \\ \texttt{succ}-1 & \texttt{x}-7 & \texttt{y}-6, \\ \texttt{succ}-2 & \texttt{x}-1 & \texttt{y}-2, \\ \texttt{succ}-3 & \texttt{x}-2 & \texttt{y}-2, \\ \texttt{succ}-2 & \texttt{x}-5 & \texttt{y}-3, \\ \texttt{succ}-3 & \texttt{x}-8 & \texttt{y}-2, \\ \texttt{succ}-9 & \texttt{x}-6 & \texttt{y}-2, \\ \texttt{succ}-10 & \texttt{x}-10 & \texttt{y}-6, \\ \texttt{succ}-8 & \texttt{x}-10 & \texttt{y}-1 \end{array} \right\rangle\right)$$

Figure 5.280 shows the line-segments associated with the NODES collection. One can note the following line-segments intersection:

- Arcs $8 \rightarrow 9$ and $7 \rightarrow 3$ cross,
- Arcs $5 \rightarrow 3$ and $6 \rightarrow 2$ cross also.

Consequently, the graph_crossing constraint holds since its first argument NCROSS is set to 2.

Typical

$|\texttt{NODES}| > 1$
$\texttt{range}(\texttt{NODES.succ}) > 1$
$\texttt{range}(\texttt{NODES.x}) > 1$
$\texttt{range}(\texttt{NODES.y}) > 1$

Symmetries

- Attributes of NODES are permutable w.r.t. permutation (succ) (x, y) (*permutation applied to all items*).
- One and the same constant can be added to the x attribute of all items of NODES.
- One and the same constant can be added to the y attribute of all items of NODES.

Usage

This is a general crossing constraint that can be used in conjunction with one graph covering constraint such as cycle, tree or map. In many practical problems ones want not only to cover a graph with specific patterns but also to avoid too much crossing between the arcs of the final graph.

Remark

We did not give a specific crossing constraint for each graph covering constraint. We feel that it is better to start first with a more general constraint before going in the specificity of the pattern that is used for covering the graph.

See also

common keyword: crossing *(line-segments intersection)*, cycle, map, tree *(graph constraint,graph partitioning constraint)*, two_layer_edge_crossing *(line-segments intersection)*.

Keywords

constraint type: graph constraint, graph partitioning constraint.

geometry: geometrical constraint, line-segments intersection.

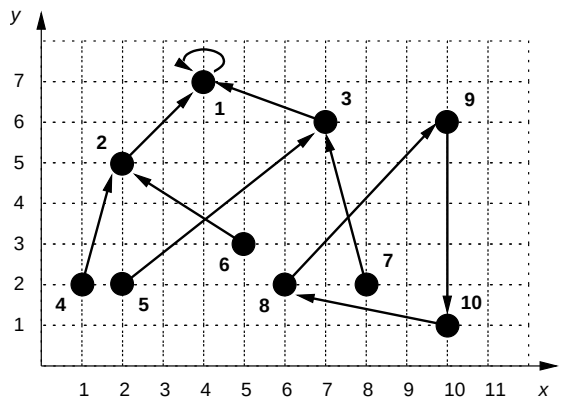


Figure 5.280: A graph covering with 2 line-segments intersections

Arc input(s) NODES

Arc generator $CLIQUE(<) \mapsto \texttt{collection}(\texttt{n1}, \texttt{n2})$

Arc arity 2

Arc constraint(s)

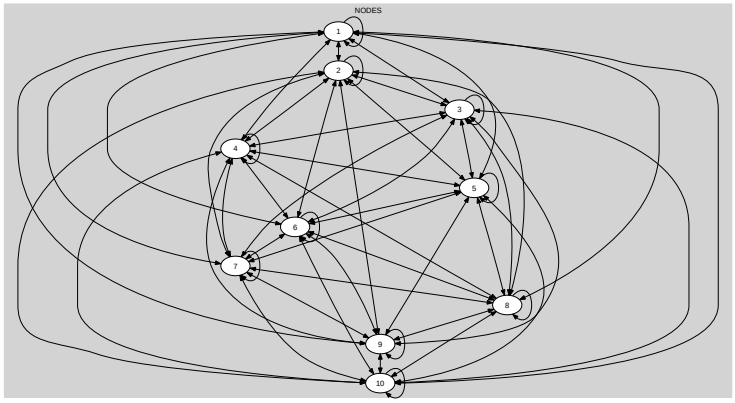
- $\texttt{max}(\texttt{n1.x}, \texttt{NODES}[\texttt{n1.succ}].\texttt{x}) \geq \texttt{min}(\texttt{n2.x}, \texttt{NODES}[\texttt{n2.succ}].\texttt{x})$
- $\texttt{max}(\texttt{n2.x}, \texttt{NODES}[\texttt{n2.succ}].\texttt{x}) \geq \texttt{min}(\texttt{n1.x}, \texttt{NODES}[\texttt{n1.succ}].\texttt{x})$
- $\texttt{max}(\texttt{n1.y}, \texttt{NODES}[\texttt{n1.succ}].\texttt{y}) \geq \texttt{min}(\texttt{n2.y}, \texttt{NODES}[\texttt{n2.succ}].\texttt{y})$
- $\texttt{max}(\texttt{n2.y}, \texttt{NODES}[\texttt{n2.succ}].\texttt{y}) \geq \texttt{min}(\texttt{n1.y}, \texttt{NODES}[\texttt{n1.succ}].\texttt{y})$
- $\begin{array}{l}(\texttt{n2.x} - \texttt{NODES}[\texttt{n1.succ}].\texttt{x}) * (\texttt{NODES}[\texttt{n1.succ}].\texttt{y} - \texttt{n1.y}) - \\ (\texttt{NODES}[\texttt{n1.succ}].\texttt{x} - \texttt{n1.x}) * (\texttt{n2.y} - \texttt{NODES}[\texttt{n1.succ}].\texttt{y})\end{array} \neq 0$
- $\begin{array}{l}(\texttt{NODES}[\texttt{n2.succ}].\texttt{x} - \texttt{NODES}[\texttt{n1.succ}].\texttt{x}) * (\texttt{n2.y} - \texttt{n1.y}) - \\ (\texttt{n2.x} - \texttt{n1.x}) * (\texttt{NODES}[\texttt{n2.succ}].\texttt{y} - \texttt{NODES}[\texttt{n1.succ}].\texttt{y})\end{array} \neq 0$
- $\begin{array}{l}\texttt{sign}\left(\begin{array}{l}(\texttt{n2.x} - \texttt{NODES}[\texttt{n1.succ}].\texttt{x}) * (\texttt{NODES}[\texttt{n1.succ}].\texttt{y} - \texttt{n1.y}) - \\ (\texttt{NODES}[\texttt{n1.succ}].\texttt{x} - \texttt{n1.x}) * (\texttt{n2.y} - \texttt{NODES}[\texttt{n1.succ}].\texttt{y})\end{array}\right) \neq \\ \texttt{sign}\left(\begin{array}{l}(\texttt{NODES}[\texttt{n2.succ}].\texttt{x} - \texttt{NODES}[\texttt{n1.succ}].\texttt{x}) * (\texttt{n2.y} - \texttt{n1.y}) - \\ (\texttt{n2.x} - \texttt{n1.x}) * (\texttt{NODES}[\texttt{n2.succ}].\texttt{y} - \texttt{NODES}[\texttt{n1.succ}].\texttt{y})\end{array}\right)\end{array}$

Graph property(ies) **NARC**= NCROSS

Graph model Each node is described by its coordinates x and y, and by its successor succ in the final covering. Note that the co-ordinates are initially fixed. We use the arc generator $CLIQUE(<)$ in order to avoid counting twice the same line-segment crossing.

Parts (A) and (B) of Figure 5.281 respectively show the initial and final graph associated with the **Example** slot. Since we use the **NARC** graph property, the arcs of the final graph are stressed in bold. Each arc of the final graph corresponds to a proper intersection between two line-segments.

(A)



(B)

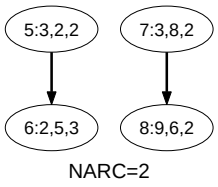


Figure 5.281: Initial and final graph of the `graph_crossing` constraint