

5.139 global_cardinality_low_up

	DESCRIPTION	LINKS	GRAPH
Origin	Used for defining <code>sliding_distribution</code> .		
Constraint	<code>global_cardinality_low_up(VARIABLES, VALUES)</code>		
Synonyms	<code>gcc_low_up</code> , <code>gcc</code> .		
Arguments	VARIABLES : <code>collection(var-dvar)</code> VALUES : <code>collection(val-int, omin-int, omax-int)</code>		
Restrictions	<pre> required(VARIABLES, var) VALUES > 0 required(VALUES, [val, omin, omax]) distinct(VALUES, val) VALUES.omin ≥ 0 VALUES.omax ≤ VARIABLES VALUES.omin ≤ VALUES.omax </pre>		
Purpose	<div style="border: 1px solid pink; padding: 5px;"> Each value <code>VALUES[i].val</code> ($1 \leq i \leq VALUES$) should be taken by at least <code>VALUES[i].omin</code> and at most <code>VALUES[i].omax</code> variables of the <code>VARIABLES</code> collection. </div>		
Example	<div style="border: 1px solid blue; padding: 10px; display: inline-block;"> $\left(\begin{array}{l} \langle 3, 3, 8, 6 \rangle, \\ \left\langle \begin{array}{lll} \text{val} - 3 & \text{omin} - 2 & \text{omax} - 3, \\ \text{val} - 5 & \text{omin} - 0 & \text{omax} - 1, \\ \text{val} - 6 & \text{omin} - 1 & \text{omax} - 2 \end{array} \right\rangle \end{array} \right)$ </div> <p>The <code>global_cardinality_low_up</code> constraint holds since values 3, 5 and 6 are respectively used 2 ($2 \leq 2 \leq 3$), 0 ($0 \leq 0 \leq 1$) and 1 ($1 \leq 1 \leq 2$) times within the collection <code>\langle 3, 3, 8, 6 \rangle</code> and since no constraint was specified for value 8.</p>		
Typical	<pre> VARIABLES > 1 range(VARIABLES.var) > 1 VALUES > 1 VALUES.omin ≤ VARIABLES VALUES.omax > 0 VALUES.omax ≤ VARIABLES VARIABLES > VALUES </pre>		

Symmetries

- Items of VARIABLES are [permutable](#).
- An occurrence of a value of VARIABLES.var that does not belong to VALUES.val can be [replaced](#) by any other value that also does not belong to VALUES.val.
- Items of VALUES are [permutable](#).
- VALUES.omin can be [decreased](#) to any value ≥ 0 .
- VALUES.omax can be [increased](#) to any value $\leq |\text{VARIABLES}|$.
- All occurrences of two distinct values in VARIABLES.var or VALUES.val can be [swapped](#); all occurrences of a value in VARIABLES.var or VALUES.val can be [renamed](#) to any unused value.

Remark

Within the context of linear programming [190, page 376] provides relaxations of the `global_cardinality_low_up` constraint.

Algorithm

A filtering algorithm achieving [arc-consistency](#) for the `global_cardinality_low_up` constraint is given in [309].

The `global_cardinality_low_up` constraint is [entailed](#) if and only if for each value v equal to `VALUES[i].val` (with $1 \leq i \leq |\text{VALUES}|$) the following two conditions hold:

1. The number of variables of the VARIABLES collection assigned value v is greater than or equal to `VALUES[i].omin`.
2. The number of variables of the VARIABLES collection that can potentially be assigned value v is less than or equal to `VALUES[i].omax`.

Reformulation

A reformulation of the `global_cardinality_low_up`, involving linear constraints, preserving [bound-consistency](#) was introduced in [65]. For each potential interval $[l, u]$ of consecutive values this model uses $|\text{VARIABLES}|$ 0-1 variables $B_{1,l,u}, B_{2,l,u}, \dots, B_{|\text{VARIABLES}|,l,u}$ for modelling the fact that each variable of the collection VARIABLES is assigned a value within interval $[l, u]$ (i.e., $\forall i \in [1, |\text{VARIABLES}|] : B_{i,l,u} \Leftrightarrow l \leq \text{VARIABLES}[i].\text{var} \wedge \text{VARIABLES}[i].\text{var} \leq u$), as well as one domain variable $C_{l,u}$ for counting how many values of $[l, u]$ are assigned to variables of VARIABLES (i.e. $C_{l,u} = B_{1,l,u} + B_{2,l,u} + \dots + B_{|\text{VARIABLES}|,l,u}$). The lower and upper bounds of variable $C_{l,u}$ are respectively initially set with respect to the minimum and maximum number of possible occurrences of the values of interval $[l, u]$. Finally, assuming that s is the smallest value that can be assigned to the variables of VARIABLES, the constraint $C_{s,u} = C_{s,k} + C_{k+1,u}$ is stated for each $k \in [s, u - 1]$.

Systems

`globalCardinality` in **Choco**.

Used in

[sliding_distribution](#).

See also

common keyword: [open_global_cardinality](#) (*assignment, counting constraint*).

generalisation: [global_cardinality](#) (*fixed interval replaced by variable*).

implied by: [increasing_global_cardinality](#) (*a `global_cardinality_low_up` constraint where the variables are increasing*), [same_and_global_cardinality_low_up](#).

related: [ordered_global_cardinality](#) (*restrictions are done on nested sets of values, all starting from first value*).

shift of concept: `global_cardinality_low_up_no_loop` (assignment of a variable to its position is ignored).

soft variant: `open_global_cardinality_low_up` (a set variable defines the set of variables that are actually considered).

specialisation: `alldifferent` (each value should occur at most once).

system of constraints: `sliding_distribution` (one `global_cardinality_low_up` constraint for each sliding sequence of SEQ consecutive variables).

Keywords

application area: assignment.

constraint type: value constraint, counting constraint.

filtering: flow, arc-consistency, bound-consistency, DFS-bottleneck.

	For all items of VALUES:
Arc input(s)	VARIABLES
Arc generator	$SELF \mapsto collection(variables)$
Arc arity	1
Arc constraint(s)	$variables.var = VALUES.val$
Graph property(ies)	<ul style="list-style-type: none"> • NVERTEX \geq VALUES.omin • NVERTEX \leq VALUES.omax

Graph model

Since we want to express one unary constraint for each value we use the “For all items of VALUES” iterator. Part (A) of Figure 5.271 shows the initial graphs associated with each value 3, 5 and 6 of the VALUES collection of the **Example** slot. Part (B) of Figure 5.271 shows the two corresponding final graphs respectively associated with values 3 and 6 that are both assigned to the variables of the VARIABLES collection (since value 5 is not assigned to any variable of the VARIABLES collection the final graph associated with value 5 is empty). Since we use the **NVERTEX** graph property, the vertices of the final graphs are stressed in bold.

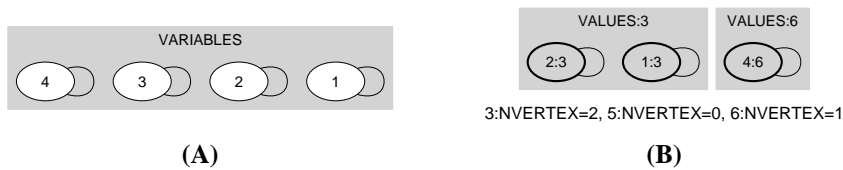


Figure 5.271: Initial and final graph of the `global_cardinality_low_up` constraint