

5.125 elements_alldifferent

	DESCRIPTION	LINKS	GRAPH
Origin	Derived from <code>elements</code> and <code>alldifferent</code> .		
Constraint	<code>elements_alldifferent</code> (ITEMS, TABLE)		
Synonyms	<code>elements_alldiff</code> , <code>elements_alldistinct</code> .		
Arguments	ITEMS : <code>collection</code> (index-dvar, value-dvar) TABLE : <code>collection</code> (index-int, value-dvar)		
Restrictions	<pre> required(ITEMS, [index, value]) ITEMS.index ≥ 1 ITEMS.index ≤ TABLE ITEMS = TABLE required(TABLE, [index, value]) TABLE.index ≥ 1 TABLE.index ≤ TABLE distinct(TABLE, index) </pre>		
Purpose	All the items of the ITEMS collection should be equal to one of the entries of the table TABLE and all the variables ITEMS.index should take distinct values.		
Example	$\left(\left\langle \begin{array}{ll} \text{index-2} & \text{value-9,} \\ \text{index-1} & \text{value-6,} \\ \text{index-4} & \text{value-9,} \\ \text{index-3} & \text{value-2} \end{array} \right\rangle, \left\langle \begin{array}{ll} \text{index-1} & \text{value-6,} \\ \text{index-2} & \text{value-9,} \\ \text{index-3} & \text{value-2,} \\ \text{index-4} & \text{value-9} \end{array} \right\rangle \right)$		
	The <code>elements_alldifferent</code> constraint holds since, as depicted by Figure 5.256, there is a one to one correspondence between the items of the ITEMS collection and the items of the TABLE collection.		
	<div style="display: flex; justify-content: space-around; margin-top: 5px;"> ITEMS TABLE </div>		

Figure 5.256: Illustration of the one to one correspondence between the items of ITEMS and the items of TABLE

Typical

```
|ITEMS| > 1
range(ITEMS.value) > 1
|TABLE| > 1
range(TABLE.value) > 1
```

Symmetries

- Arguments are [permutable](#) w.r.t. permutation (ITEMS, TABLE).
- Items of ITEMS are [permutable](#).
- Items of TABLE are [permutable](#).
- All occurrences of two distinct values in ITEMS.value or TABLE.value can be [swapped](#); all occurrences of a value in ITEMS.value or TABLE.value can be [renamed](#) to any unused value.

Usage

Used for replacing by one single `elements_alldifferent` constraint an `alldifferent` and a set of `element` constraints having the following structure:

- The union of the index variables of the `element` constraints is equal to the set of variables of the `alldifferent` constraint.
- All the `element` constraints share exactly the same table.

For instance, the constraint given in the **Example** slot is equivalent to the conjunction of the following set of constraints:

$$\text{alldifferent}(\langle \text{var} - 2, \text{var} - 1, \text{var} - 4, \text{var} - 3 \rangle)$$

$$\text{element} \left(\begin{array}{l} \langle \text{index} - 2 \quad \text{value} - 9 \rangle, \\ \text{index} - 1 \quad \text{value} - 6, \\ \langle \text{index} - 2 \quad \text{value} - 9, \\ \text{index} - 3 \quad \text{value} - 2, \\ \text{index} - 4 \quad \text{value} - 9 \rangle \end{array} \right)$$

$$\text{element} \left(\begin{array}{l} \langle \text{index} - 1 \quad \text{value} - 6 \rangle, \\ \text{index} - 1 \quad \text{value} - 6, \\ \langle \text{index} - 2 \quad \text{value} - 9, \\ \text{index} - 3 \quad \text{value} - 2, \\ \text{index} - 4 \quad \text{value} - 9 \rangle \end{array} \right)$$

$$\text{element} \left(\begin{array}{l} \langle \text{index} - 3 \quad \text{value} - 2 \rangle, \\ \text{index} - 1 \quad \text{value} - 6, \\ \langle \text{index} - 2 \quad \text{value} - 9, \\ \text{index} - 3 \quad \text{value} - 2, \\ \text{index} - 4 \quad \text{value} - 9 \rangle \end{array} \right)$$

$$\text{element} \left(\begin{array}{l} \langle \text{index} - 4 \quad \text{value} - 9 \rangle, \\ \text{index} - 1 \quad \text{value} - 6, \\ \langle \text{index} - 2 \quad \text{value} - 9, \\ \text{index} - 3 \quad \text{value} - 2, \\ \text{index} - 4 \quad \text{value} - 9 \rangle \end{array} \right)$$

As a practical example of utilisation of the `elements_alldifferent` constraint we show how to model the link between a permutation consisting of one single cycle and its expanded form. For instance, to the permutation 3, 6, 5, 2, 4, 1 corresponds the sequence 3 5 4 2 6 1. Let us note $S_1, S_2, S_3, S_4, S_5, S_6$ the permutation and $V_1 V_2 V_3 V_4 V_5 V_6$ its expanded form (see Figure 5.257).

The constraint:

$$\text{elements_alldifferent} \left(\begin{array}{l} \text{index} - V_1 \quad \text{value} - V_2, \\ \text{index} - V_2 \quad \text{value} - V_3, \\ \left\langle \begin{array}{l} \text{index} - V_3 \quad \text{value} - V_4, \\ \text{index} - V_4 \quad \text{value} - V_5, \\ \text{index} - V_5 \quad \text{value} - V_6, \\ \text{index} - V_6 \quad \text{value} - V_1 \end{array} \right\rangle, \\ \text{index} - 1 \quad \text{value} - S_1, \\ \text{index} - 2 \quad \text{value} - S_2, \\ \left\langle \begin{array}{l} \text{index} - 3 \quad \text{value} - S_3, \\ \text{index} - 4 \quad \text{value} - S_4, \\ \text{index} - 5 \quad \text{value} - S_5, \\ \text{index} - 6 \quad \text{value} - S_6 \end{array} \right\rangle \end{array} \right)$$

models the fact that $S_1, S_2, S_3, S_4, S_5, S_6$ corresponds to a permutation with one single cycle. It also expresses the link between the variables $S_1, S_2, S_3, S_4, S_5, S_6$ and $V_1, V_2, V_3, V_4, V_5, V_6$.

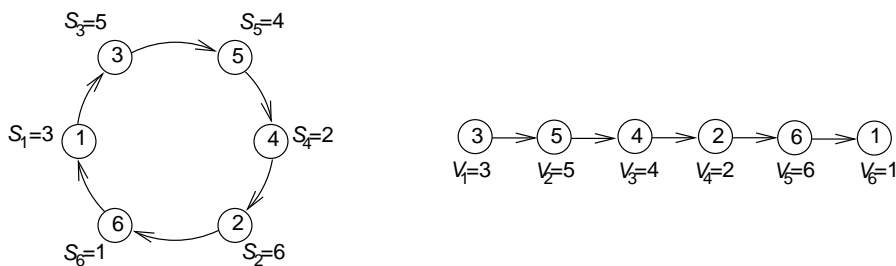


Figure 5.257: Two representations of a permutation containing one single cycle

Reformulation

The `elements_alldifferent`(`<index - I_1 value - V_1 , index - I_2 value - $V_2, \dots, \text{index} - I_{|ITEMS|}$ value - $V_{|ITEMS|}$ >`, TABLE) constraint can be expressed in term of a conjunction of `|ITEMS| elem` constraints and of one `alldifferent` constraint of the form:

```
elem(<index -  $I_1$  value -  $V_1$ >, TABLE),
elem(<index -  $I_2$  value -  $V_2$ >, TABLE),
...
elem(<index -  $I_{|ITEMS|}$  value -  $V_{|ITEMS|}$ >, TABLE),
alldifferent(<< $I_1, I_2, \dots, I_{|ITEMS|}$ >>).
```

See also

[implies: elements.](#)

[used in reformulation: alldifferent, elem, element.](#)

Keywords

characteristic of a constraint: disequality.

combinatorial object: permutation.

constraint type: data constraint.

modelling: array constraint, table, functional dependency.

Arc input(s)	ITEMS TABLE
Arc generator	<i>PRODUCT</i> \mapsto collection(items, table)
Arc arity	2
Arc constraint(s)	<ul style="list-style-type: none"> • items.index = table.index • items.value = table.value
Graph property(ies)	<u>NVERTEX</u> = ITEMS + TABLE

Graph model

The fact that all variables ITEMS.index are pairwise different is derived from the conjunctions of the following facts:

- From the graph property $\text{NVERTEX} = |\text{ITEMS}| + |\text{TABLE}|$ it follows that all vertices of the initial graph belong also to the final graph,
- A vertex v belongs to the final graph if there is at least one constraint involving v that holds,
- From the first condition items.index = table.index of the arc constraint, and from the restriction distinct(TABLE.index) it follows: for all vertices v generated from the collection ITEMS at most one constraint involving v holds.

Parts (A) and (B) of Figure 5.258 respectively show the initial and final graph associated with the **Example** slot. Since we use the NVERTEX graph property, the vertices of the final graph are stressed in bold.

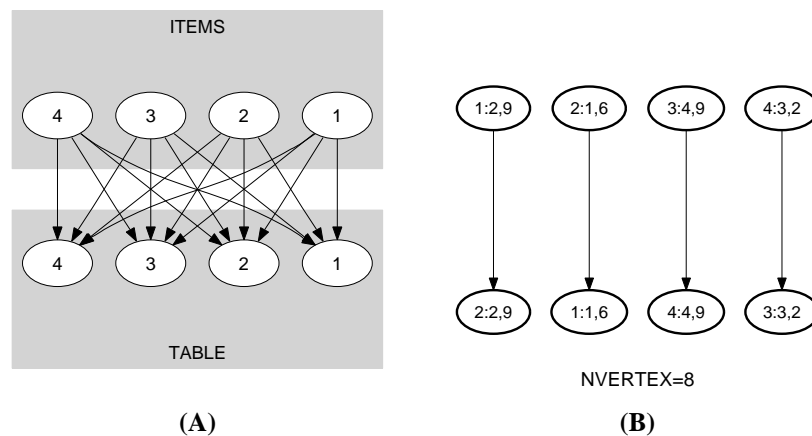


Figure 5.258: Initial and final graph of the `elements_alldifferent` constraint

Signature

Since the final graph cannot have more than $|\text{ITEMS}| + |\text{TABLE}|$ vertices one can simplify NVERTEX to NVERTEX.

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