

5.116 elem_from_to

	DESCRIPTION	LINKS	AUTOMATON
Origin	Derived from <code>elem</code> .		
Constraint	<code>elem_from_to</code> (ITEM, TABLE)		
Synonym	<code>element_from_to</code> .		
Arguments	<pre> ITEM : collection (from-dvar, cst_from-int, to-dvar, cst_to-int, value-dvar) TABLE : collection(index-int, value-dvar) </pre>		
Restrictions	<pre> required(ITEM, [from, cst_from, to, cst_to, value]) ITEM.from ≥ 1 ITEM.from ≤ TABLE ITEM.to ≥ 1 ITEM.to ≤ TABLE ITEM.from ≤ ITEM.to ITEM = 1 required(TABLE, [index, value]) TABLE.index ≥ 1 TABLE.index ≤ TABLE increasing_seq(TABLE, [index]) </pre>		
Purpose	<p>Let FROM, CST_FROM, TO, CST_TO, VALUE respectively denote the attributes <code>ITEM[1].from</code>, <code>ITEM[1].cst_from</code>, <code>ITEM[1].to</code>, <code>ITEM[1].cst_to</code>, <code>ITEM[1].value</code> of the unique item of the ITEM collection.</p> <p>Beside imposing the fact that $FROM \leq TO$ and that both FROM and TO are assigned a value in $[1, TABLE]$, the <code>elem_from_to</code> constraint enforces the following condition: All entries of the TABLE collection from position $\max(1, FROM + CST_FROM)$ to position $\min(TABLE , TO + CST_TO)$ are equal to VALUE. When $\max(1, FROM + CST_FROM)$ is strictly greater than $\min(TABLE , TO + CST_TO)$ the constraint holds no matter what value is assigned to VALUE.</p>		
Example	$\left(\begin{array}{l} \langle \text{from} - 1 \text{ cst_from} - 1 \text{ to} - 4 \text{ cst_to} - -1 \text{ value} - 2 \rangle, \\ \text{index} - 1 \quad \text{value} - 6, \\ \langle \text{index} - 2 \quad \text{value} - 2, \\ \text{index} - 3 \quad \text{value} - 2, \rangle \\ \text{index} - 4 \quad \text{value} - 9, \\ \text{index} - 5 \quad \text{value} - 9 \end{array} \right)$ <p>The <code>elem_from_to</code> constraint holds since all entries between position $\max(1, FROM + CST_FROM) = \max(1, 1 + 1) = 2$ and position $\min(TABLE , TO + CST_TO) = \min(5, 4 - 1) = 3$ are equal to 2.</p>		

Typical

```

ITEM.cst_from ≥ 0
ITEM.cst_from ≤ 1
ITEM.cst_to ≥ -1
ITEM.cst_to ≤ 1
|TABLE| > 1
range(TABLE.value) > 1

```

Symmetry

All occurrences of two distinct values in `ITEM.value` or `TABLE.value` can be [swapped](#); all occurrences of a value in `ITEM.value` or `TABLE.value` can be [renamed](#) to any unused value.

Usage

Given an array $t[1..n]$ of integers (i.e., an array of integers for which the entries are defined between 1 and n), the `elem_from_to` constraint is for instance useful for encoding expressions of the form $\exists i \in [1, n], \forall j \in [i + 1, n] \mid t[i] = 0$. Note that, when the interval $[i + 1, n]$ is empty, the condition $\forall j \in [i + 1, n] \mid t[i] = 0$ is satisfied and i is equal to n . This example is encoded by using an `elem_from_to` constraint and by respectively setting:

- FROM to i , where i is a variable that is assigned a value from interval $[1, n]$,
- CST_FROM to constant 1,
- TO to n , the index of the last entry of the array $t[1..n]$,
- CST_TO to constant 0,
- VALUE to 0, the value we are looking for.
- TABLE to the array of integers $t[1..n]$.

Finally, note that j is not used at all.

See also

[common keyword](#): `elem`, `element` (*array constraint*).

Keywords

[characteristic of a constraint](#): `automaton`, `automaton without counters`, `reified automaton constraint`.

[constraint type](#): `data constraint`.

[filtering](#): `arc-consistency`.

[modelling](#): `array constraint`, `table`, `variable indexing`, `variable subscript`.

Automaton

Figure 5.235 depicts the automaton associated with the `elem_from_to` constraint.

Let us first introduce some notations:

- Let n denote the number of items of the TABLE collection.
- Let $INDEX_i$ and $VALUE_i$ respectively be the index and the value attributes of the i^{th} item of the TABLE collection.
- Let $FROM$, CST_FROM , TO , CST_TO , $VALUE$ respectively denote the attributes $ITEM[1].from$, $ITEM[1].cst_from$, $ITEM[1].to$, $ITEM[1].cst_to$, $ITEM[1].value$ of the unique item of the ITEM collection.
- Let IN be a shortcut for condition $1 \leq FROM \wedge FROM \leq TO \wedge TO \leq n$.
- Let F and T respectively denote the quantities $\max(1, FROM + CST_FROM)$ and $\min(|TABLE|, TO + CST_TO)$.

To each septuple $(FROM, TO, F, T, VALUE, INDEX_i, VALUE_i)$ corresponds a signature variable S_i as well as the following signature constraint:

$$\left\{ \begin{array}{ll} (IN \wedge F > T) & \Leftrightarrow S_i = 0 \wedge \\ (IN \wedge F \leq T \wedge F > INDEX_i) & \Leftrightarrow S_i = 1 \wedge \\ (IN \wedge F \leq T \wedge T < INDEX_i) & \Leftrightarrow S_i = 2 \wedge \\ (IN \wedge F \leq T \wedge F \leq INDEX_i \wedge INDEX_i \leq T \wedge VALUE = VALUE_i) & \Leftrightarrow S_i = 3 \wedge \\ (IN \wedge F \leq T \wedge F \leq INDEX_i \wedge INDEX_i \leq T \wedge VALUE \neq VALUE_i) & \Leftrightarrow S_i = 4 \end{array} \right.$$

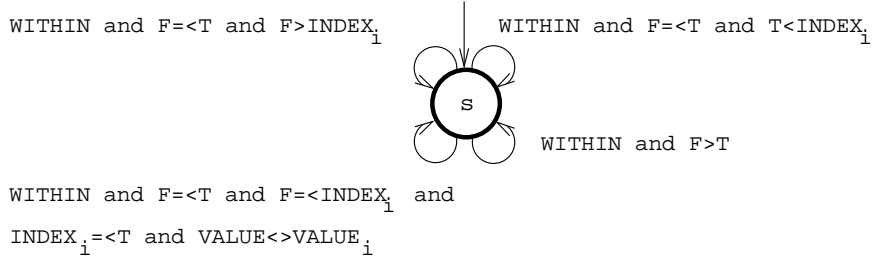


Figure 5.235: Automaton of the `elem_from_to` constraint

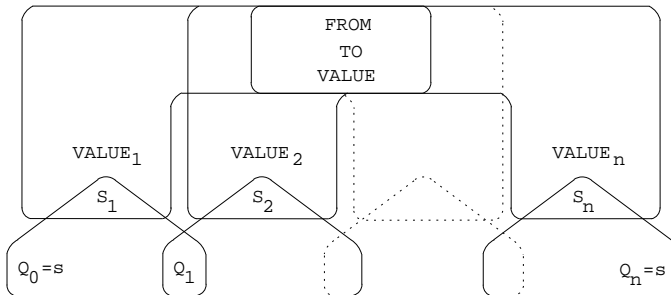


Figure 5.236: Hypergraph of the reformulation corresponding to the automaton of the `elem_from_to` constraint

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