5.81 crossing

DESCRIPTION LINKS GRAPH

Origin Inspired by [110].

Constraint crossing(NCROSS, SEGMENTS)

Arguments

NCROSS	:	dvar
SEGMENTS	:	collection(ox−dvar, oy−dvar, ex−dvar, ey−dvar)

Restrictions

$\texttt{NCROSS} \geq 0$
$\texttt{NCROSS} \leq (|\texttt{SEGMENTS}| * |\texttt{SEGMENTS}| - |\texttt{SEGMENTS}|)/2$
required(SEGMENTS, [ox, oy, ex, ey])

Purpose NCROSS is the number of line-segments intersections between the line-segments defined by the SEGMENTS collection. Each line-segment is defined by the coordinates (ox, oy) and (ex, ey) of its two extremities.

Example

$$\left(3, \left\langle \begin{array}{cccc} \texttt{ox}-1 & \texttt{oy}-4 & \texttt{ex}-9 & \texttt{ey}-2, \\ \texttt{ox}-1 & \texttt{oy}-1 & \texttt{ex}-3 & \texttt{ey}-5, \\ \texttt{ox}-3 & \texttt{oy}-2 & \texttt{ex}-7 & \texttt{ey}-4, \\ \texttt{ox}-9 & \texttt{oy}-1 & \texttt{ex}-9 & \texttt{ey}-4 \end{array} \right\rangle\right)$$

Figure 5.163 provides a picture of the example with the corresponding four line-segments of the SEGMENTS collection. The crossing constraint holds since its first argument NCROSS is set to 3, which is actually the number of line-segments intersections.

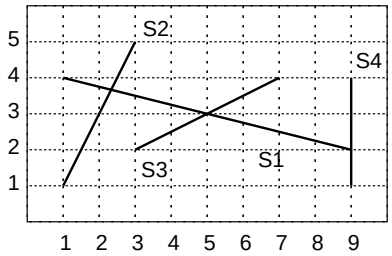

Figure 5.163: Intersection between line-segments

Typical $|\texttt{SEGMENTS}| > 1$

Symmetries

- Items of SEGMENTS are permutable.
- Attributes of SEGMENTS are permutable w.r.t. permutation (ox, oy) (ex, ey) (*permutation applied to all items*).
- One and the same constant can be added to the ox and ex attributes of all items of SEGMENTS.
- One and the same constant can be added to the oy and ey attributes of all items of SEGMENTS.

See also **common keyword:** graph_crossing, two_layer_edge_crossing *(line-segments intersection)*.

Keywords **final graph structure:** acyclic, no loop.

geometry: geometrical constraint, line-segments intersection.

Arc input(s) SEGMENTS

Arc generator $CLIQUE(<) \mapsto \texttt{collection}(\texttt{s1}, \texttt{s2})$

Arc arity 2

Arc constraint(s)

- $\texttt{max}(\texttt{s1.ox}, \texttt{s1.ex}) \geq \texttt{min}(\texttt{s2.ox}, \texttt{s2.ex})$
- $\texttt{max}(\texttt{s2.ox}, \texttt{s2.ex}) \geq \texttt{min}(\texttt{s1.ox}, \texttt{s1.ex})$
- $\texttt{max}(\texttt{s1.oy}, \texttt{s1.ey}) \geq \texttt{min}(\texttt{s2.oy}, \texttt{s2.ey})$
- $\texttt{max}(\texttt{s2.oy}, \texttt{s2.ey}) \geq \texttt{min}(\texttt{s1.oy}, \texttt{s1.ey})$
- $$\bigvee \left(\begin{array}{l} (\texttt{s2.ox} - \texttt{s1.ex}) * (\texttt{s1.ey} - \texttt{s1.oy}) - \prod \left(\begin{array}{l} \texttt{s1.ex} - \texttt{s1.ox}, \\ \texttt{s2.oy} - \texttt{s1.ey} \end{array} \right) = 0, \\ (\texttt{s2.ex} - \texttt{s1.ex}) * (\texttt{s2.oy} - \texttt{s1.oy}) - \prod \left(\begin{array}{l} \texttt{s2.ox} - \texttt{s1.ox}, \\ \texttt{s2.ey} - \texttt{s1.ey} \end{array} \right) = 0, \\ \texttt{sign} \left(\begin{array}{l} (\texttt{s2.ox} - \texttt{s1.ex}) * (\texttt{s1.ey} - \texttt{s1.oy}) - \\ (\texttt{s1.ex} - \texttt{s1.ox}) * (\texttt{s2.oy} - \texttt{s1.ey}) \end{array} \right) \neq \\ \texttt{sign} \left(\begin{array}{l} (\texttt{s2.ex} - \texttt{s1.ex}) * (\texttt{s2.oy} - \texttt{s1.oy}) - \\ (\texttt{s2.ox} - \texttt{s1.ox}) * (\texttt{s2.ey} - \texttt{s1.ey}) \end{array} \right) \end{array} \right)$$

Graph property(ies) NARC= NCROSS

Graph class

- ACYCLIC
- NO_LOOP

Graph model Each line-segment is described by the x and y coordinates of its two extremities. In the arc generator we use the restriction $<$ in order to generate one single arc for each pair of segments. This is required, since otherwise we would count more than once a given line-segments intersection.

Parts (A) and (B) of Figure 5.164 respectively show the initial and final graph associated with the **Example** slot. Since we use the NARC graph property, the arcs of the final graph are stressed in bold. An arc constraint expresses the fact the two line-segments intersect. It is taken from [110, page 889]. Each arc of the final graph corresponds to a line-segments intersection.

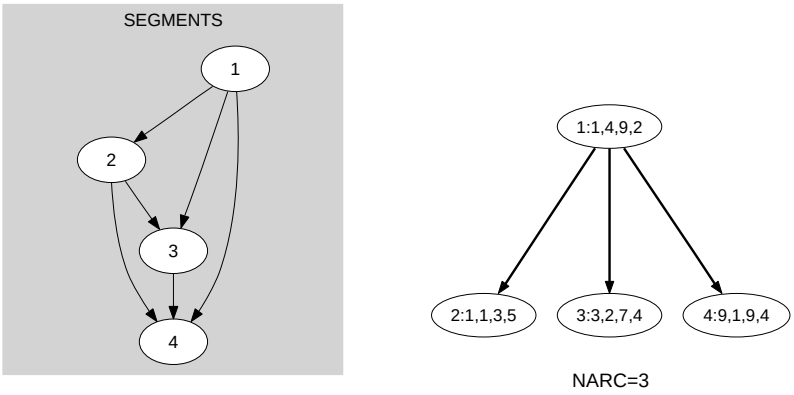

Figure 5.164: Initial and final graph of the `crossing` constraint