5.50 change_continuity

DESCRIPTION LINKS GRAPH AUTOMATON

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Constraint

$$\texttt{change\_continuity}\left(\begin{array}{l}\texttt{NB\_PERIOD\_CHANGE},\\ \texttt{NB\_PERIOD\_CONTINUITY},\\ \texttt{MIN\_SIZE\_CHANGE},\\ \texttt{MAX\_SIZE\_CHANGE},\\ \texttt{MIN\_SIZE\_CONTINUITY},\\ \texttt{MAX\_SIZE\_CONTINUITY},\\ \texttt{NB\_CHANGE},\\ \texttt{NB\_CONTINUITY},\\ \texttt{VARIABLES},\\ \texttt{CTR}\end{array}\right)$$

Arguments

NB_PERIOD_CHANGE	:	dvar
NB_PERIOD_CONTINUITY	:	dvar
MIN_SIZE_CHANGE	:	dvar
MAX_SIZE_CHANGE	:	dvar
MIN_SIZE_CONTINUITY	:	dvar
MAX_SIZE_CONTINUITY	:	dvar
NB_CHANGE	:	dvar
NB_CONTINUITY	:	dvar
VARIABLES	:	collection(var−dvar)
CTR	:	atom

Restrictions

NB_PERIOD_CHANGE ≥ 0
NB_PERIOD_CONTINUITY ≥ 0
MIN_SIZE_CHANGE ≥ 0
MAX_SIZE_CHANGE $\geq$ MIN_SIZE_CHANGE
MIN_SIZE_CONTINUITY ≥ 0
MAX_SIZE_CONTINUITY $\geq$ MIN_SIZE_CONTINUITY
NB_CHANGE ≥ 0
NB_CONTINUITY ≥ 0
required(VARIABLES, var)
CTR $\in [=, \neq, <, \geq, >, \leq]$

On the one hand a *change* is defined by the fact that constraint $\texttt{VARIABLES}[i].\texttt{var}\ \texttt{CTR}\ \texttt{VARIABLES}[i+1].\texttt{var}$ holds.
On the other hand a *continuity* is defined by the fact that constraint $\texttt{VARIABLES}[i].\texttt{var}\ \texttt{CTR}\ \texttt{VARIABLES}[i+1].\texttt{var}$ does not hold.
A *period of change* on variables

$$\texttt{VARIABLES}[i].\texttt{var}, \texttt{VARIABLES}[i+1].\texttt{var}, \ldots, \texttt{VARIABLES}[j].\texttt{var}\ (i < j)$$

is defined by the fact that all constraints $\texttt{VARIABLES}[k].\texttt{var}\ \texttt{CTR}\ \texttt{VARIABLES}[k+1].\texttt{var}$ hold for $k \in [i, j-1]$.
A *period of continuity* on variables

$$\texttt{VARIABLES}[i].\texttt{var}, \texttt{VARIABLES}[i+1].\texttt{var}, \ldots, \texttt{VARIABLES}[j].\texttt{var}\ (i < j)$$

is defined by the fact that all constraints $\texttt{VARIABLES}[k].\texttt{var}\ \texttt{CTR}\ \texttt{VARIABLES}[k+1].\texttt{var}$ do not hold for $k \in [i, j-1]$.

Purpose

The constraint `change_continuity` holds if and only if:

- `NB_PERIOD_CHANGE` is equal to the number of periods of change,
- `NB_PERIOD_CONTINUITY` is equal to the number of periods of continuity,
- `MIN_SIZE_CHANGE` is equal to the number of variables of the smallest period of change,
- `MAX_SIZE_CHANGE` is equal to the number of variables of the largest period of change,
- `MIN_SIZE_CONTINUITY` is equal to the number of variables of the smallest period of continuity,
- `MAX_SIZE_CONTINUITY` is equal to the number of variables of the largest period of continuity,
- `NB_CHANGE` is equal to the total number of changes,
- `NB_CONTINUITY` is equal to the total number of continuities.

Example

$$\left(\begin{array}{c} \\ \\ \\ \\ 3,2,2,4,2,4,6,4,\left\langle \begin{array}{c} \texttt{var}-1, \\ \texttt{var}-3, \\ \texttt{var}-1, \\ \texttt{var}-8, \\ \texttt{var}-8, \\ \texttt{var}-4, \\ \texttt{var}-7, \\ \texttt{var}-7, \\ \texttt{var}-7, \\ \texttt{var}-7, \\ \texttt{var}-2 \end{array} \right\rangle, \neq \\ \\ \\ \\ \end{array}\right)$$

Figure 5.95 makes clear the different parameters that are associated with the given example for the collection $\texttt{VARIABLES} = \langle 1,3,1,8,8,4,7,7,7,7,2 \rangle$. We place character | for representing a change and a blank for a continuity. On top of the solution we represent the different periods of change, while below we show the different periods of continuity. The `change_continuity` constraint holds since:

- Its number of periods of change NB_PERIOD_CHANGE is equal to 3 (i.e., the 3 periods depicted on top of Figure 5.95),
- Its number of periods of continuity NB_PERIOD_CONTINUITY is equal to 2 (i.e., the 2 periods depicted below Figure 5.95),
- The number of variables of its smallest period of change MIN_SIZE_CHANGE is equal to 2 (i.e., the number of variables involved in the third period of change 7 2 depicted on top of Figure 5.95),
- The number of variables of the largest period of change MAX_SIZE_CHANGE is equal to 4 (i.e., the number of variables involved in the first period of change 1 3 1 8 depicted on top of Figure 5.95),
- The number of variables of the smallest period of continuity MIN_SIZE_CONTINUITY is equal to 2 (i.e., the number of variables involved in the first period 8 8 depicted below Figure 5.95),
- The number of variables of the largest period of continuity MAX_SIZE_CONTINUITY is equal to 4 (i.e., the number of variables involved in the second period 7 7 7 7 depicted below Figure 5.95),
- The total number of changes NB_CHANGE is equal to 6 (i.e., the number of occurrences of character | in Figure 5.95),
- The total number of continuities NB_CONTINUITY is equal to 4.

```
<-----> <---->     <->
1|3|1|8 8|4|7 7 7 7|2
      <->   <----->
```

Figure 5.95: Periods of changes and periods of continuities

Typical

NB_PERIOD_CHANGE > 0
NB_PERIOD_CONTINUITY > 0
MIN_SIZE_CHANGE > 0
MIN_SIZE_CONTINUITY > 0
NB_CHANGE > 0
NB_CONTINUITY > 0
$|$VARIABLES$| > 1$
range(VARIABLES.var) > 1

Symmetry

One and the same constant can be added to the var attribute of all items of VARIABLES.

Remark

If the variables of the collection VARIABLES have to take distinct values between 1 and the total number of variables, we have what is called a permutation. In this case, if we choose the binary constraint $<$, then MAX_SIZE_CHANGE gives the size of the longest run of the permutation; A *run* is a maximal increasing contiguous subsequence in a permutation.

See also

common keyword: group, group_skip_isolated_item, stretch_path *(timetabling constraint)*.

Keywords

characteristic of a constraint: automaton, automaton with counters.

combinatorial object: sequence, run of a permutation, permutation.

constraint network structure: sliding cyclic(1) constraint network(2), sliding cyclic(1) constraint network(3).

constraint type: timetabling constraint.

final graph structure: connected component, apartition, acyclic, bipartite, no loop.

miscellaneous: obscure.

Arc input(s)	VARIABLES
Arc generator	$PATH \mapsto$ collection(variables1, variables2)
Arc arity	2
Arc constraint(s)	variables1.var CTR variables2.var
Graph property(ies)	• NCC = NB_PERIOD_CHANGE • MIN_NCC = MIN_SIZE_CHANGE • MAX_NCC = MAX_SIZE_CHANGE • NARC = NB_CHANGE
Graph class	• ACYCLIC • BIPARTITE • NO_LOOP

Arc input(s)	VARIABLES
Arc generator	$PATH \mapsto$ collection(variables1, variables2)
Arc arity	2
Arc constraint(s)	variables1.var$\neg$ CTR variables2.var
Graph property(ies)	• NCC = NB_PERIOD_CONTINUITY • MIN_NCC = MIN_SIZE_CONTINUITY • MAX_NCC = MAX_SIZE_CONTINUITY • NARC = NB_CONTINUITY
Graph class	• ACYCLIC • BIPARTITE • NO_LOOP

Graph model We use two graph constraints to respectively catch the constraints on the period of changes and of the period of continuities. In both case each period corresponds to a connected component of the final graph.

Parts (A) and (B) of Figure 5.96 respectively show the initial and final graph associated with the first graph constraint of the **Example** slot.

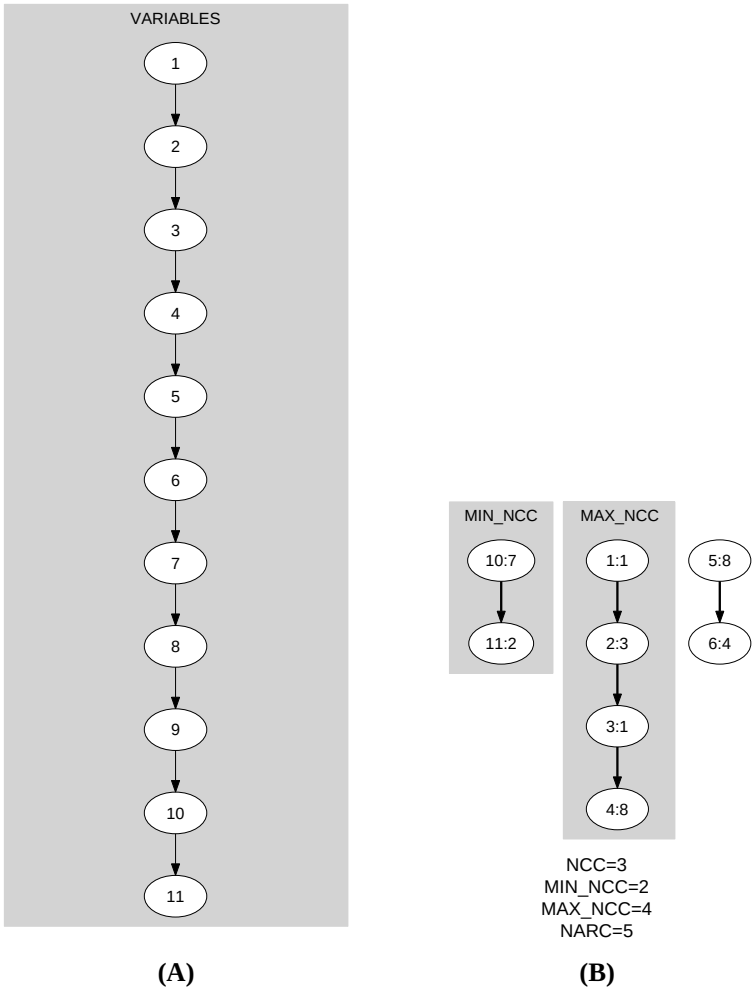

Figure 5.96: Initial and final graph of the `change_continuity` constraint

Automaton

Figures 5.97 , 5.98 , 5.101 , 5.102 , 5.105 , 5.106 and 5.109 depict the automata associated with the different graph parameters of the change_continuity constraint. For the automata that respectively compute NB_PERIOD_CHANGE, NB_PERIOD_CONTINUITY MIN_SIZE_CHANGE, MIN_SIZE_CONTINUITY MAX_SIZE_CHANGE, MAX_SIZE_CONTINUITY NB_CHANGE and NB_CONTINUITY we have a 0-1 signature variable $\mathtt{S}_i$ for each pair of consecutive variables $(\mathtt{VAR}_i, \mathtt{VAR}_{i+1})$ of the collection VARIABLES. The following signature constraint links $\mathtt{VAR}_i$, $\mathtt{VAR}_{i+1}$ and $\mathtt{S}_i$: $\mathtt{VAR}_i$ CTR $\mathtt{VAR}_{i+1} \Leftrightarrow \mathtt{S}_i$.

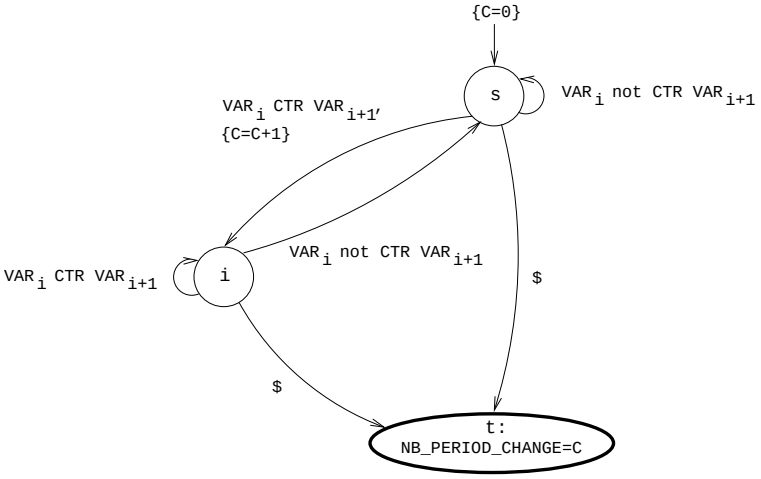

Figure 5.97: Automaton for the NB_PERIOD_CHANGE parameter of the change_continuity constraint

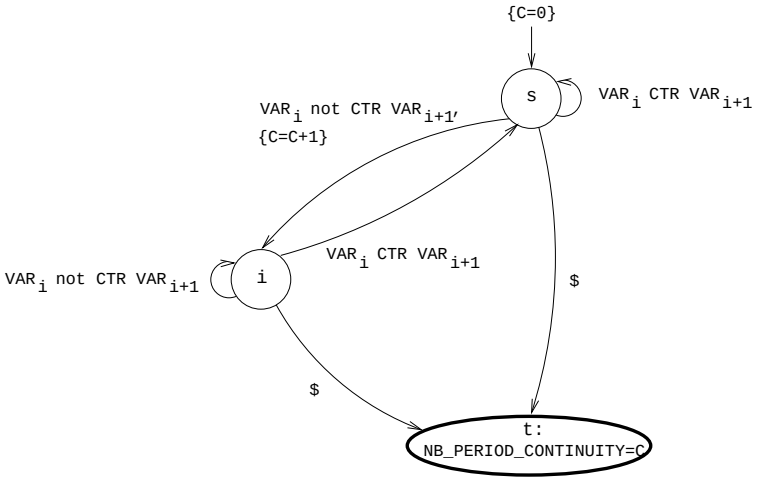

Figure 5.98: Automaton for the NB_PERIOD_CONTINUITY parameter of the change_continuity constraint

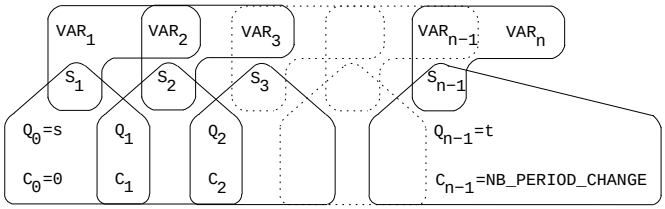

Figure 5.99: Hypergraph of the reformulation corresponding to the automaton of the NB_PERIOD_CHANGE parameter of the change_continuity constraint

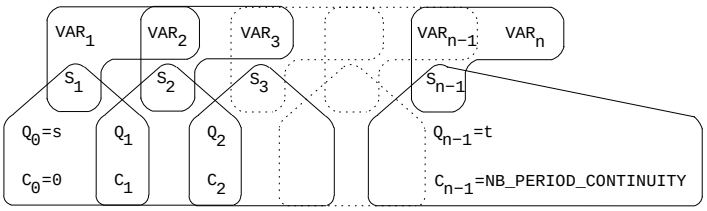

Figure 5.100: Hypergraph of the reformulation corresponding to the automaton of the NB_PERIOD_CONTINUITY parameter of the change_continuity constraint

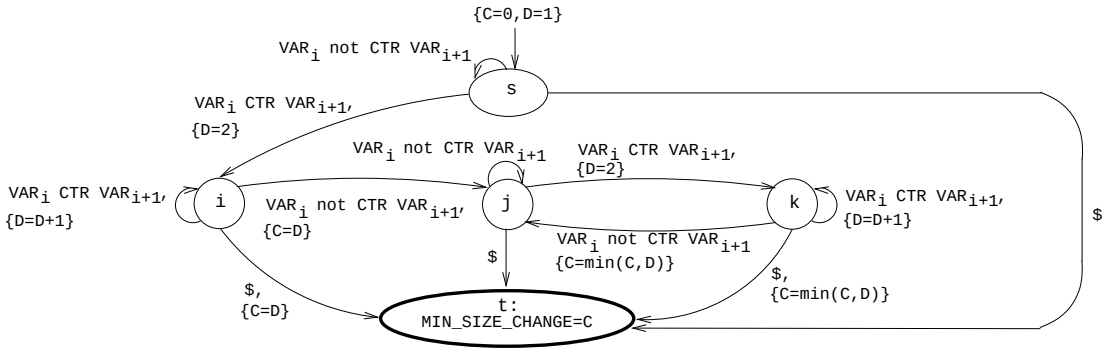

Figure 5.101: Automaton for the MIN_SIZE_CHANGE parameter of the change_continuity constraint

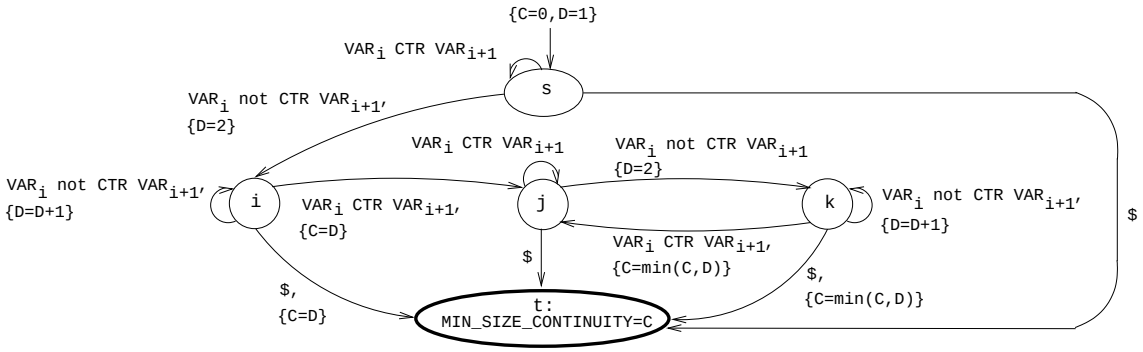

Figure 5.102: Automaton for the MIN_SIZE_CONTINUITY parameter of the change_continuity constraint

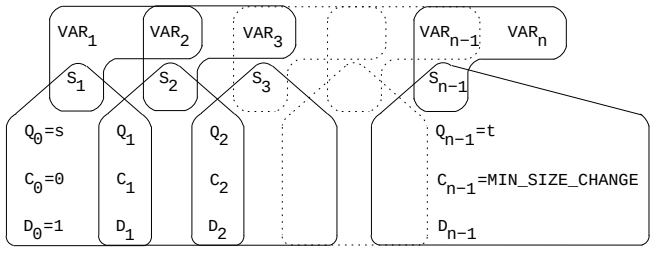

Figure 5.103: Hypergraph of the reformulation corresponding to the automaton of the MIN_SIZE_CHANGE parameter of the change_continuity constraint

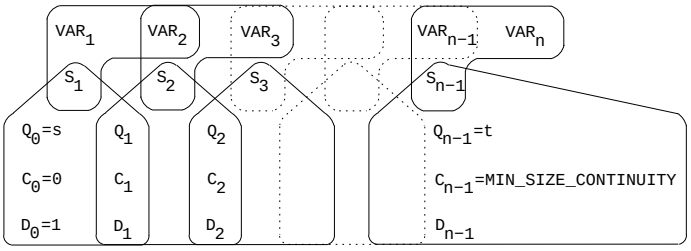

Figure 5.104: Hypergraph of the reformulation corresponding to the automaton of the MIN_SIZE_CONTINUITY parameter of the change_continuity constraint

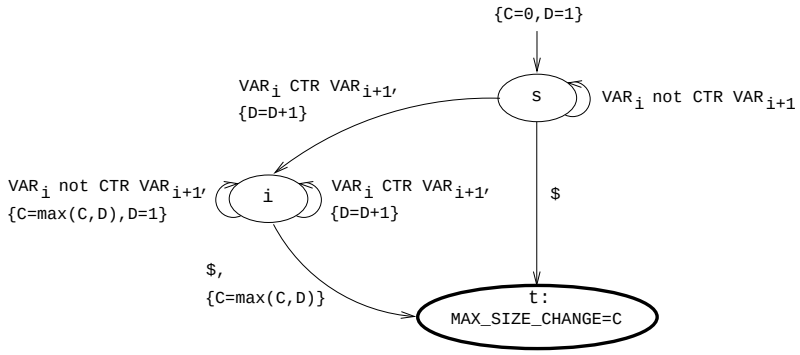

Figure 5.105: Automaton for the MAX_SIZE_CHANGE parameter of the change_continuity constraint

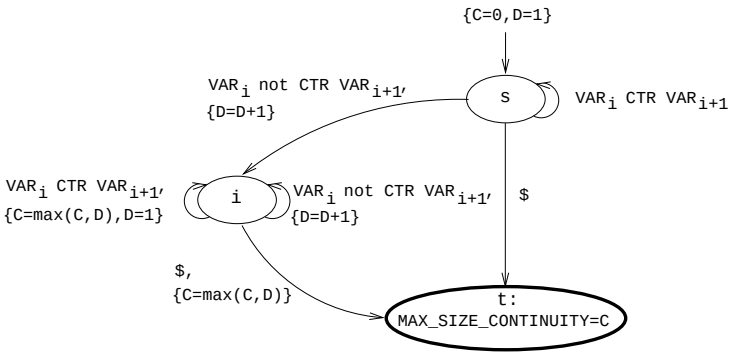

Figure 5.106: Automaton for the MAX_SIZE_CONTINUITY parameter of the change_continuity constraint

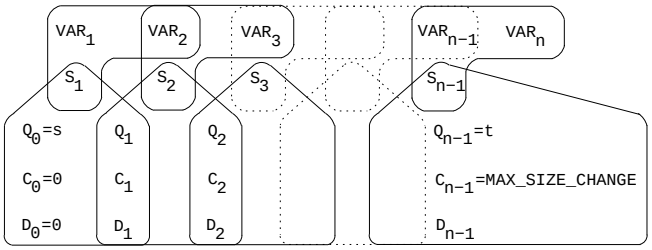

Figure 5.107: Hypergraph of the reformulation corresponding to the automaton of the MAX_SIZE_CHANGE parameter of the change_continuity constraint

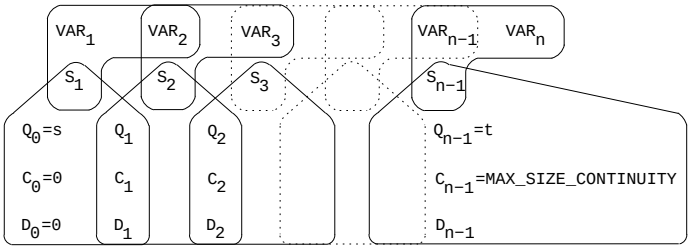

Figure 5.108: Hypergraph of the reformulation corresponding to the automaton of the MAX_SIZE_CONTINUITY parameter of the change_continuity constraint

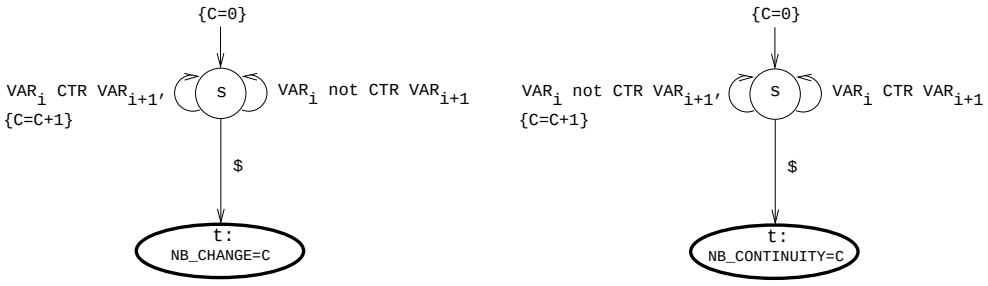

Figure 5.109: Automata for the NB_CHANGE and NB_CONTINUITY parameters of the change_continuity constraint

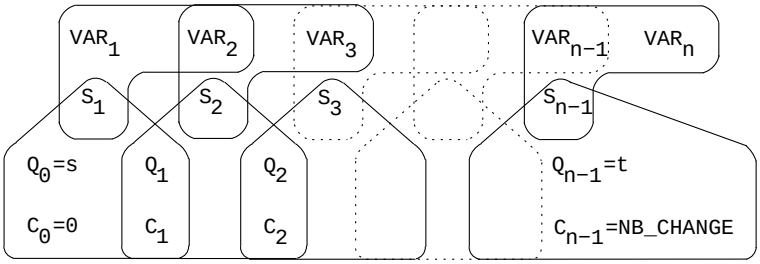

Figure 5.110: Hypergraph of the reformulation corresponding to the automaton of the NB_CHANGE parameter of the change_continuity constraint

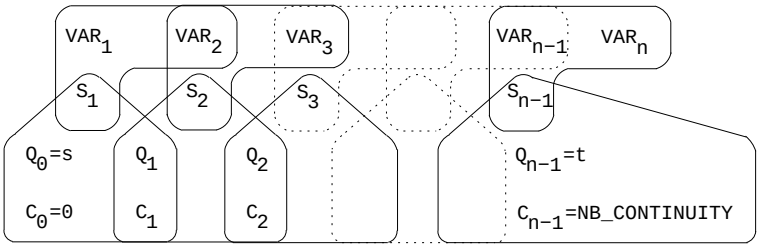

Figure 5.111: Hypergraph of the reformulation corresponding to the automaton of the NB_CONTINUITY parameter of the change_continuity constraint