5.27 assign_and_counts

DESCRIPTION LINKS GRAPH AUTOMATON

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Constraint assign_and_counts(COLOURS, ITEMS, RELOP, LIMIT)

Arguments

COLOURS	:	collection(val−int)
ITEMS	:	collection(bin−dvar, colour−dvar)
RELOP	:	atom
LIMIT	:	dvar

Restrictions

required(COLOURS, val)
distinct(COLOURS, val)
required(ITEMS, [bin, colour])
RELOP $\in [=, \neq, <, \geq, >, \leq]$

Purpose Given several items (each of them having a specific colour that may not be initially fixed), and different bins, assign each item to a bin, so that the total number n of items of colour COLOURS in each bin satisfies the condition n RELOP LIMIT.

Example

$$\left(\begin{array}{l} \langle 4 \rangle, \\ \left\langle \begin{array}{ll} \texttt{bin} - 1 & \texttt{colour} - 4, \\ \texttt{bin} - 3 & \texttt{colour} - 4, \\ \texttt{bin} - 1 & \texttt{colour} - 4, \\ \texttt{bin} - 1 & \texttt{colour} - 5 \end{array} \right\rangle, \leq, 2 \end{array} \right)$$

Figure 5.50 shows the solution associated with the example. The items and the bins are respectively represented by little squares and by the different columns. Each little square contains the value of the key attribute of the item to which it corresponds. The items for which the colour attribute is equal to 4 are located under the thick line. The assign_and_counts constraint holds since for each used bin (i.e., namely bins 1 and 3) the number of assigned items for which the colour attribute is equal to 4 is less than or equal to the limit 2.

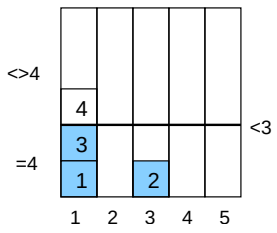


Figure 5.50: Assignment of the items to the bins

Typical

$|\texttt{COLOURS}| > 0$
$|\texttt{ITEMS}| > 1$
$\texttt{range}(\texttt{ITEMS.bin}) > 1$
$\texttt{LIMIT} > 0$
$\texttt{LIMIT} < |\texttt{ITEMS}|$

Symmetries

- Items of COLOURS are permutable.
- Items of ITEMS are permutable.
- All occurrences of two distinct values of ITEMS.bin can be swapped; all occurrences of a value of ITEMS.bin can be renamed to any unused value.

Usage

Some persons have pointed out that it is impossible to use constraints such as among, atleast, atmost, count, or global_cardinality if the set of variables is not initially known. However, this is for instance required in practice for some timetabling problems.

See also

assignment dimension removed: count, counts.

used in graph description: counts.

Keywords

application area: assignment.

characteristic of a constraint: coloured, automaton, automaton with array of counters, derived collection.

final graph structure: acyclic, bipartite, no loop.

modelling: assignment dimension.

Derived Collection

col(VALUES−collection(val−int), [item(val − COLOURS.val)])

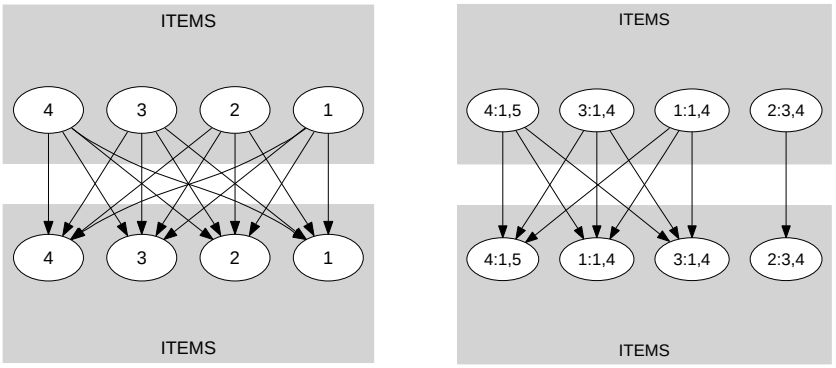
Arc input(s)	ITEMS ITEMS
Arc generator	$PRODUCT \mapsto$ collection(items1, items2)
Arc arity	2
Arc constraint(s)	items1.bin = items2.bin
Graph class	• ACYCLIC • BIPARTITE • NO_LOOP
Sets	SUCC $\mapsto$ [source, variables − col(VARIABLES−collection(var−dvar), [item(var − ITEMS.colour)])]
Constraint(s) on sets	counts(VALUES, variables, RELOP, LIMIT)

Graph model

We enforce the counts constraint on the colour of the items that are assigned to the same bin.

Parts (A) and (B) of Figure 5.51 respectively show the initial and final graph associated with the **Example** slot. The final graph consists of the following two connected components:

- The connected component containing six vertices corresponds to the items that are assigned to bin 1.
- The connected component containing two vertices corresponds to the items that are assigned to bin 3.



(A) **(B)**

Figure 5.51: Initial and final graph of the assign_and_counts constraint

The assign_and_counts constraint holds since for each set of successors of the vertices of the final graph no more than two items take colour 4.

Automaton

Figure 5.52 depicts the automaton associated with the `assign_and_counts` constraint. To each `colour` attribute $\texttt{COLOUR}_i$ of the collection `ITEMS` corresponds a 0-1 signature variable $\texttt{S}_i$. The following signature constraint links $\texttt{COLOUR}_i$ and $\texttt{S}_i$: $\texttt{COLOUR}_i \in \texttt{COLOURS} \Leftrightarrow \texttt{S}_i$. For all items of the collection `ITEMS` for which the `colour` attribute takes its value in `COLOURS`, counts for each value assigned to the `bin` attribute its number of occurrences n, and finally imposes the condition n `RELOP LIMIT`.

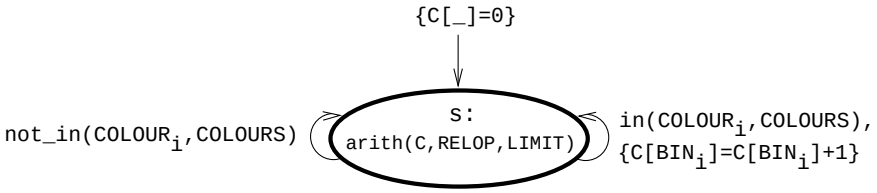


Figure 5.52: Automaton of the `assign_and_counts` constraint