Computing capture tubes

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1 Introduction

A dynamic system can often be described by a state equation $\dot{\mathbf{x}} = \mathbf{h}(\mathbf{x}, \mathbf{u}, t)$ where $\mathbf{x} \in \mathbb{R}^n$ is the state vector, $\mathbf{u} \in \mathbb{R}^m$ is the control vector and $\mathbf{h} : \mathbb{R}^n \times \mathbb{R}^p \times \mathbb{R} \to \mathbb{R}^n$ is the evolution function. Assume that the control low $\mathbf{u} = \mathbf{g}(\mathbf{x}, t)$ is known (this can be obtained using control theory), the system becomes autonomous. If we define $\mathbf{f}(\mathbf{x}, t) = \mathbf{h}(\mathbf{x}, \mathbf{g}(\mathbf{x}, t), t)$, we get the following equation.

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t).$$

The validation of some stability properties of this system is an important and difficult problem [2] which can be transformed into proving the inconsistency of a constraint satisfaction problem. For some particular properties and for invariant system (i.e., $\mathbf{f}$ does not depend on t), it has been shown [1] that the V-stability approach combined interval analysis [3] can solve the problem efficiently. Here, we extend this work to systems where $\mathbf{f}$ depends on time.

2 Problem statement

Consider an autonomous system described by a state equation $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t)$. A *tube* $\mathbb{G}(t)$ is a function which associates to each $t \in \mathbb{R}$ a subset of $\mathbb{R}^n$. A tube $\mathbb{G}(t)$ is said to be a *capture tube* if the fact that $\mathbf{x}(t) \in \mathbb{G}(t)$ implies that $\mathbf{x}(t + t_1) \in \mathbb{G}(t + t_1)$ for all $t_1 > 0$. Consider the tube

$$\mathbb{G}(t) = \{\mathbf{x}, \mathbf{g}(\mathbf{x}, t) \le \mathbf{0}\} \tag{1}$$

where $\mathbf{g} : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^m$. The following theorem, introduced recenltly [4], shows that the problem of proving that $\mathbb{G}(t)$ is a capture tube can be cast into solving a set of inequalities.

Theorem. If the system of constraints

$$\begin{cases} \text{(i)} & \frac{\partial g_i}{\partial \mathbf{x}}(\mathbf{x},t).\mathbf{f}(\mathbf{x},t) + \frac{\partial g_i}{\partial t}(\mathbf{x},t) \geq 0 \\ \text{(ii)} & g_i(\mathbf{x},t) = 0 \\ \text{(iii)} & \mathbf{g}(\mathbf{x},t) \leq 0 \end{cases} \quad (2)$$

is inconsistent for all $\mathbf{x}$, all $t \geq 0$ and all $i \in \{1,\ldots,m\}$ then $\mathbb{G}(t) = \{\mathbf{x}, \mathbf{g}(\mathbf{x},t) \leq \mathbf{0}\}$ is a capture tube.

3 Computing capture tubes

If a candidate $\mathbb{G}(t)$ for a capture tube is available, we can check that $\mathbb{G}(t)$ is a capture tube by checking the inconsistency of a set of nonlinear equations (see the previous section). This inconsistency can then easily be checked using interval analysis [3]. Now, for many systems such as for non holonomous systems, we rarely have a candidate for a capture tube and we need to find one. Our main contribution is to provide a method that can help us to find such a capture tube. The idea if to start from a non-capture tube $\mathbb{G}(t)$ and to try to characterize the smallest capture tube $\mathbb{G}^+(t)$ which encloses $\mathbb{G}(t)$. To do this, we predict for all $(\mathbf{x}, t)$, that are solutions of (2), a guaranteed envelope for trajectory within finite time-horizon window $[t, t+t_2]$ (where $t_2 > 0$ is fixed). If all corresponding $\mathbf{x}(t+t_2)$ belongs to $\mathbb{G}(t+t_2)$, then the union of all trajectories and the initial $\mathbb{G}(t)$ (in the (x,t) space) corresponds to the smallest capture tube enclosing $\mathbb{G}(t)$.

References

[1] L. jaulin, F. Le Bars, An interval approach for stability analysis; Application to sailboat robotics, *IEEE Transaction on Robotics*, 2012.

[2] S. Le Menec, Linear Differential Game with Two Pursuers and One Evader, *Advances in Dynamic Games*, 2011.

[3] R.E. Moore, R.B. Kearfott, M.J. Cloud, *Introduction to Interval Analysis*, SIAM, Philadelphia, 2009.

[4] A. Stancu, L. Jaulin, A. Bethencourt, *Set-membership tracking using capture tubes*, to be submitted.